RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. THIRD SEMESTER EXAMINATION, DECEMBER 2015

SECOND YEAR [BATCH 2014-17]

Date : 23/12/2015 **MATHEMATICS [General]**

Time : 11 am – 2 pm **Paper :** III Full Marks : 75

[Use a separate Answer Book for each group]

Group – A

(*Answer **any four** questions*)

1. If P be the point (2, 3, –1), then find the equation of the plane through P at right angles to the straight line OP where O is the origin. [5]

2. Find the equation of the plane bisecting the angle between the planes $3x-6y+2z+5=0$ and $4x-12y+3z-3=0$ which contains the origin. [5]

3. Find the shortest distance between the straight lines $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$ and $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$. Find also the equations of the line of shortest distance. [3+2]

4. Show that only one tangent plane can be drawn to the sphere $x^2+y^2+z^2-2x+6y+2z+8=0$ through the line $3x-4y-8=0,\ y-3z+2=0$. [5]

5. Find the equation of the sphere for which the circle $x^2+y^2+z^2=9,\ x+y+z+3=0$ is a great circle. [5]

6. Find the equation of the right circular cone whose vertex is the origin and whose axis is the line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and semi-vertical angle is $\frac{\pi}{3}$. [5]

Group – B

(*Answer **any four** questions*)

7. a) A company is considering two types of buses, 'type A' and 'type B' for transportation. A 'type A' bus can carry 40 passengers daily and requires 2 machines for servicing, a 'type B' bus can carry 60 passengers daily and requires 5 machines for servicing. The company must transport at least 300 person daily and not more than 12 machines can be employed. The cost of purchasing buses is to be minimised, given that a 'type A' bus costs Rs. 1,20,000 and 'type B' bus costs Rs. 1,50,000. Formulate this problem as an L.P.P. [4]

   b) Is the solution (2,1,3) of the system
   $4x_1+2x_2-3x_3=1$
   $6x_1+4x_2-5x_3=1$
   a basic feasible solution? If not then reduce it to a basic feasible solution. [6]

8. a) Solve the following L.P.P graphically :
   Maximise z = 2x + 5y
   subject to the constraints $0\le x\le 4,\ 0\le y\le 3$ and $x+y\le 6$. [5]

   b) Prove that the set of all feasible solutions of an L.P.P is a convex set. [5]

9. a) Find all the basic feasible solutions of the system
   $x_1+2x_3=1$
   $x_2+x_3=4$. [6]

   b) If x_1, x_2 be real, then show that the set X given by $X=\{(x_1,x_2)\,|\,x_1+2x_2\le 5\}$ is a convex set. [4]

10. a) Solve the following L.P.P by simplex method :

Minimize $z = 3x_1 - 2x_2$

Subject to $x_1 - x_2 \le 1$

$3x_1 - 2x_2 \le 6$

and $x_1, x_2 \ge 0$ [4]

b) Write down the dual of the following primal problem :

Maximize $z = x_1 + 4x_2 + 3x_3$

Subject to $2x_1 + 3x_2 - 5x_3 \le 2$

$3x_1 - x_2 + 6x_3 \ge 1$

$x_1 + x_2 + x_3 = 4$

$x_1, x_2 \ge 0, x_3$ is unrestricted in sign. [6]

11. a) Use Charne's Big-M method to solve the L.P.P.

Maximize $z = 3x_1 - x_2$

Subject to $2x_1 + x_2 \ge 2$

$x_1 + 3x_2 \le 3$

$x_2 \le 4$

and $x_1, x_2 \ge 0$ [7]

b) Prove that the dual of the dual of an L.P.P is the primal. [3]

12. a) Find the initial basic feasible solution of the following Transportation Problem by North-West-Corner rule : [3]

	D_1	D_2	D_3	D_4	a_i
O_1	19	20	50	10	7
O_2	70	30	40	60	9
O_3	40	8	70	20	18
b_j	5	8	7	14	

b) Apply Vogel's Approximation Method to find the basic feasible solution of the following Transportation Problem : [7]

	D_1	D_2	D_3	D_4	a_i
O_1	5	3	6	4	30
O_2	3	4	7	8	15
O_3	9	6	5	8	15
b_j	10	25	18	7	

13. a) Find the optimal assignments to find the minimum cost for the assignment problem with the following cost matrix. [6]

	1	2	3	4
A	10	12	19	11
B	5	10	7	8
C	12	14	13	11
D	8	15	11	9

b) Write down the analytical definition of an extreme point of a convex set. Give an example to show that the boundary points of a set are not necessarily the extreme points. What can you say about the converse? [2+1+1]

Group – C

(*Answer **any three** questions*)

14. Calculate f(1·141) from the following table : [5]

x	1·140	1·145	1·150	1·155	1·160	1·165	1·170	1·175
f(x)	0·131028	0·135405	0·139762	0·144100	0·148420	0·152721	0·157004	0·161268

15. Find $\int_0^{\frac{\pi}{2}} \sqrt{\sin x}\,dx$ by Trapezoidal rule, taking 5 equal sub-intervals correct to 3 decimal places. What is the geometrical significance of Trapezoidal rule? [4+1]

16. Compute $\int_2^{10} \frac{dx}{1+x}$ using Simpson's $\frac{1}{3}$-rd rule, taking 1·0 as the length of each sub-interval, correct to four places after decimal. Hence find the absolute error. [4+1]

17. Find the real root of the equation $10^x + x - 4 = 0$ correct to 2 places of decimals lying between 0 & 1 by tabulation method. [5]

18. Establish the following relations between the shift operator 'E' and the difference operator 'Δ' :
    a) $E = \Delta + 1$ [3]
    b) $E \cdot \Delta = \Delta \cdot E$ [2]

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